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Otter versus Pufferfish: The Functional Importance of Geometry in Nature
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Otter versus Pufferfish: The Functional Importance of Geometry in Nature

Leslie Jones, Daniel Huber and Rebecca J. Waggett

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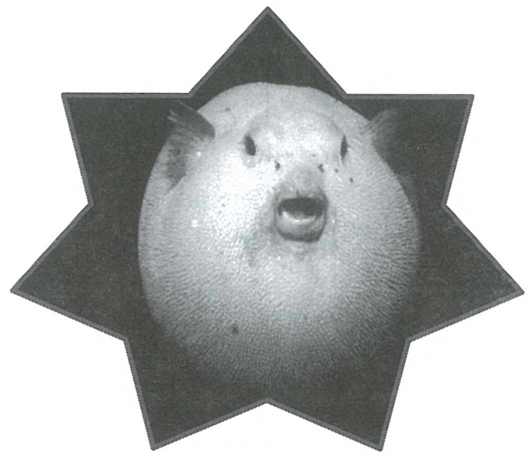

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Building rich context for lessons entices the learner into the problem by establishing relevance and purpose, which facilitates interest and memory. Geometry and biology can be natural partners in the quest to bring depth of meaning and enthusiasm into the classroom because geometry plays a fundamental role in determining organismal function. In this article, we showcase a lesson on the African clawless otter and the geometry-changing pufferfish. There are accompanying PowerPoint slides for this lesson, as well as worksheets and additional presentations for other enriching integrated lessons, on our website:

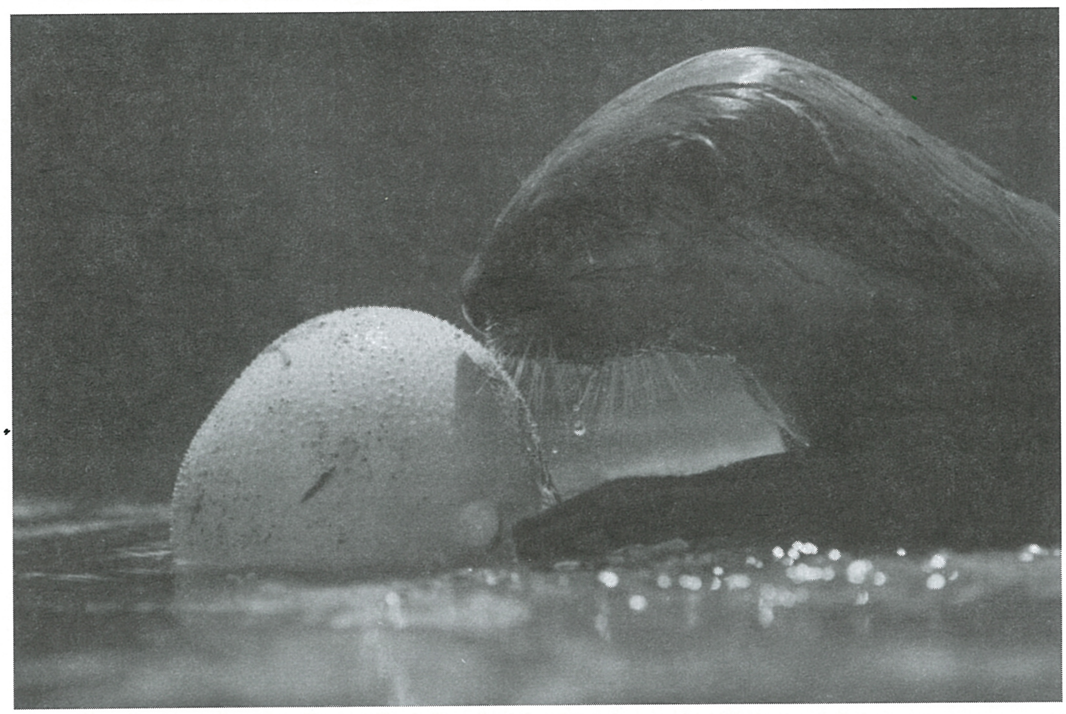
http://utweb.ut.edu/rwaggett/science-math-master.html.

The geometry of anatomical structures can play a significant role in ecological interactions among species, which ultimately determine the survival of individual organisms and contribute to the success or demise of entire species. Predation is particularly important in this regard, with mouth size being a geometric parameter that significantly affects a predator's ability to capture a given prey item. Predators with large mouths have a competitive advantage because they are able to consume larger prey, which provide a greater energetic return per unit effort than small prey. Therefore, large-mouthed predators can possibly displace small-mouthed predators from a particular ecological niche owing to the better energetic efficiency of their feeding strategy. The relationship between mouth size and predatory ability is generally straight forward; the larger the predator's mouth, the larger the prey it can eat. However, an interesting case arises when we examine prey items that are able to change size in the presence of a predator, as is the case with pufferfish. Pufferfish have evolved the ability to rapidly increase size when threatened by a predator by pumping water into an expandable stomach. This process is aided by extensible skin and the absence of ribs (Wainwright and Turingan, 1997). Pufferfish can inflate within about 15 seconds to a volume approximately three times that of their resting state (Brainerd, 1994). Further, research indicates that pufferfish can remain inflated indefinitely (Brainerd, 1994).

The inspiration for this lesson is a video clip from National Geographic involving a river otter attempting to capture an inflating pufferfish. The type of otter highlighted in this video clip is *Aonyx capensis*, commonly known as the African clawless otter. They are some of the largest otters on earth, smaller only than the sea otter and the giant otter (Estes, 1991). As their common name indicates, they are found in Africa occupying primarily freshwater habitats from Ethiopia southward, though it is absent in the Congo rainforest basin (Larivière, 2001). The African clawless otter has a powerful jaw coupled with large molars suitable for crushing crabs, which dominate their diet. In addition, the clawless otter frequently consumes a variety of frogs and many fish species. The species found in their diet varies with season and location, but typically does not include pufferfish (Larivière, 2001). This begs the question of why such a capable predator, with jaws designed to crush and destroy prey, is incapable of popping and eating a pufferfish.

The objective of this lesson is to determine the largest inflated pufferfish that an otter with jaws of a specified size can successfully consume. The otter's jaws are modeled as tangents to a circle representing the pufferfish, and it is assumed that the jaws must be tangent in order for the otter's teeth to puncture the pufferfish. The "gape angle" represents the largest angle to which the otter can open its jaws and effectively apply bite pressure. To introduce the lesson, we suggest viewing the comical video clip at:

http://news.nationalgeographic.com/news/2006/01/0127_060127_puffer_video.html.



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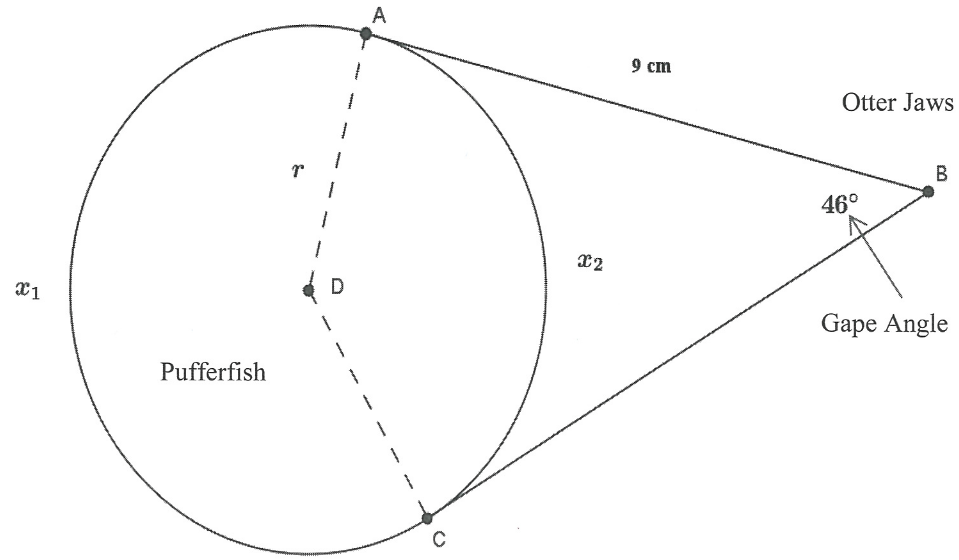
Otter versus Pufferfish Activity Sheet



1. The otter's jaws are tangent to the pufferfish. The arcs, x_1 and x_2 represent arc measurements in degrees. What is the measurement of $x_1 + x_2$?

2. Write an equation relating x_1 and x_2 to the measurement of $\angle$ABC.

3. Use your answers to the previous questions to find x_1 and x_2.

4. Draw $\overline{AC}$ and label its midpoint H. Draw $\overline{DB}$.

5. Name every triangle drawn in the picture that is similar to $\triangle$ BHA.

6. Find the length of radius, r, of the circle, to two decimal places.

7. What is the diameter of the largest pufferfish that this otter can eat?

(Answers are on page 30 – right behind the Membership EnrollmentPage)

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Dr. Rebecca Waggett is an Assistant Professor in the Biology Department at The University of Tampa. Her research areas include the biomechanics, physiology and sensory ecology of zooplankton feeding and escape behavior and the investigation of zooplankton grazer control on the development and maintenance of harmful algal blooms.

OOPS - In the last issue I made a couple of errors to the article from Ed Laughbaum. I changed the title on the cover from "Not Your Father's **Equation** Solving" to "Not Your Father's **Problem** Solving". I did the same in the header. In addition I cut off a word and a numeral in working with the text boxes. Ed has a web site which has the completely correct version for those of you who might wish to examine it further. This was an interesting article to me, and I regret the errors. No excuse other than I just got careless.

https://people.math.osu.edu/laughbaum.6/